Overdissipation:

A Reassessment of Evolutionarily Stable

Behavior in Contests

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July 2020

Abstract

The term "rent-dissipation" is prominently used in the theory of rent-seeking and its (game) theoretical vehicle contest theory. While in a pure strategy Nash equilibrium of a contest with finitely many players full rent-dissipation cannot occur, this is not the case if the same contests are analyzed with the solution concept of an evolutionarily stable strategy (ESS). Notably, even overdissipation of a rent in pure strategy ESS is feasible!

The paper explains these differences from a principle point of view by linking the ESS solution theory in contests to the Nash equilibrium solution theory of *transfer* contests. This insight then not only makes the computation of the ESS-solution of a contest from the Nash solution of a transfer contest an easy exercise, but - more importantly - also leads to a reassessment of the ESS-"overdissipation" results in the earlier

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literature: correct "rationalization" of ESS behavior in a contest for a rent as Nash behavior of absolute payoff maximizers must consider the *transfer* value instead of the value. As a consequence overdissipation cannot occur in any ESS either.

1   Introduction

Contest theory has become the (game) theoretical vehicle of rent-seeking theory. Early rent-seeking theory was grounded in the competitive paradigm: the simplest model of rent seeking hypothesizes that any artificially contrived rents will be competed away by parties seeking to secure a share of such rents. This postulate of "full rent-dissipation" was used to empirically estimate the cost of rent-seeking. It was Tullock (1980), who criticized this assumption and presented a first game-theoretic model of a contest, in which the full rent-dissipation hypothesis did not hold in Nash equilibrium. In retrospect this came as no big surprise: while the hypothesis was formulated under the tacit assumption of *perfect* competition among contestants, Tullock contests feature finitely many contestants and hence only model imperfect competition. The degree of rent-dissipation consequentially depends on and increases with the number of contestants. Underdissipation of a rent contested by rational contestants is the rule, full dissipation a limiting case (and - see below - *ex post* overdissipation is a coincidence in Nash equilibrium).

Contests are not only pervasive in economic and political life, but also in the non-human world. In fact, the very concept of a contest is rooted in biology and nature. As Congleton (2015, p. 6) states: "It bears noting that it is completely *natural* (my italics) that individuals and organizations attempt

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to profit from their efforts, as rent seekers do. It is not a human or cultural defect, but a survival trait." And, indeed, the term 'contest competition' has a history in theoretical biology before it entered economics: e.g. Van Den Berg et al. (2006, p.211) attribute it to

> "Nicholson (1954), who recognized two extreme forms of competition which he called contest and scramble competition. In contest competition, each successful competitor obtains all resources it requires for survival or reproduction; the remaining competitor, being deprived of its resources, will not be able to function anymore. In scramble competition, the finite resource is shared equally amongst the competitors so that the quantity of food per individual declines with increasing population density."

In economic terms contest competition represents all-pay winners-take-all competition, while scramble competition describes the tragedy of the commons with unregulated equal access to the resource for all. These concepts of competition turn out to be important determinants of population size and dynamics of a species. Roughly speaking, "as contest competition allows the monopolization of resources... this results in stable population dynamics, in stark contrast to scramble competition which can result in periodic or chaotic population dynamics." (Wikipedia.org; entry "contest competition"). The relevant game-theoretic methods to investigate such population behavior are those of evolutionary game theory, most notably the solution concept of an evolutionary stable strategy (ESS). An ESS comprises the stability requirements for an (unmodelled) evolutionary process, that does not rely on choice and rationality. Nevertheless in very large populations it has to be a Nash equilibrium, while not all Nash equilibria are also an ESS. ESS as

a *refinement* of Nash equilibrium, however, only applies in infinitely large populations of agents playing an evolutionary game. This in general wisdom often overlooked fact, has important consequences for the analysis of contests: as Hirshleifer (2001, p. 18) notes: "...in contrast with standard theory, conflict theory can rarely use the 'large number - perfect competition' simplification". And, indeed, an ESS of a contest with finitely many contestants is in general *not* a Nash equilibrium of the contest game (see Leininger, 2003). Moreover, *ex ante* overdissipation of the rent -while incompatible with Nash equilibrium- is possible in an ESS of a Tullock contest under increasing returns to scale in contest effort (see Hehenkamp et al., 2004).

This paper explains these differences from a principle point of view by linking the ESS solution theory in contests to the Nash equilibrium solution theory of *transfer contests*. It thereby exploits a basic result from Leininger (2003). This insight then leads to a reassessment of the ESS-"overdissipation" results in the earlier literature: as a rent in Nature cannot be "exogenously" given, but must come out of itself, the measure stick for the dissipation rate has to be adjusted. If one does so, i.e. replaces the value of the rent with its transfer value, overdissipation becomes incompatible with ESS, too. Only in a limiting case will full dissipation of the transfer value occur; just as full dissipation of the rent value only occurs as a limiting case in the Nash solution theory. The commonly known fact that ESS behavior is more "aggressive", even spiteful, than behavior in Nash equilibrium is now interpreted as survival-enhancing insurance against expropriation in the relevant *transfer* contest. In animal contests there is simply more at stake than an *exogenous* rent.

The paper is organized as follows: in section 2 we briefly review the Nash and ESS solution theory for Tullock contests with emphasis on overdissipation results. In section 3 we discuss Nash rationalizations of ESS behavior; i.e. how an ESS of a game can be understood as Nash behavior in a modified version of the game. The classical interpretation of an ESS in a finite population is that players in the contest game do not maximize absolute, but relative payoff (Schaffer, 1988, 1989). An alternative, but widely neglected interpretation is that ESS behavior refers to the Nash equilibrium of the associated transfer contest (Leininger, 2003). It is this interpretation which is the basis of the analysis of section 4, that yields a reassessment of the rent-dissipation issue in ESS. Section 5 concludes.

2   Tullock Contests

Recall that Tullock (1980) proposed a model, in which n rent-seekers compete for a rent of size V. If the contestants expend $x = (x_1, ..., x_n)$, $x_i \geq 0$, the probability of success for player i, $i = 1, ..., n$ is given by

$$p_i(x_1, ..., x_n) = \frac{x_i^r}{\sum\limits_{j=1}^{n} x_j^r}$$

and expected profit for player i is given by

$$\Pi_i(x_1, ..., x_n) = p_i(x_1, ..., x_n)V - x_i = \frac{x_i^r}{\sum\limits_{j=1}^{n} x_j^r}V - x_i.$$

For any $r \leq n/(n-1)$, a unique Nash equilibrium in pure strategies exists in this game, in which each player maximizes expected payoff by bidding

$$x^* = \frac{n-1}{n^2}rV.$$

Aggregate rent-dissipation then amounts to

$$nx^* = \frac{n-1}{n}rV.$$

Hence equilibrium expenditures never exceed V, the value of the rent, but may be strictly less than V. The "full rent dissipation"-hypothesis does not hold except in the limiting case of $r = \frac{n}{n-1}$; yet overdissipation is incompatible with individually rational payoff maximization. The utmost expenditures are reached for an arbitrarily large population ($n \to \infty$) and $r = \frac{n}{n-1}$: *almost all of V gets dissipated.*

An alternative approach to the analysis of behavior in contests is presented in Hehenkamp et al. (2004): they resort to a biologically inspired view of human and animal behavior; namely that a behavioral pattern diffuses, stabilizes, mutates and disappears along an evolutionary path that leads to the survival of best or best adapted strategies or standards of behavior. A particularly useful concept in this respect is the notion of an evolutionarily stable strategy (ESS) as defined by Maynard Smith (Maynard Smith and Price (1973); Maynard Smith (1974, 1982)), because it allows one to study stability properties of an evolutionary system without the need to commit oneself to specific dynamics.

A strategy is evolutionarily stable, if a whole population using that strategy cannot be invaded by a sufficiently small group of "mutants" using another strategy. Similarly, a standard of behavior in an economic contest is evolutionarily stable, if - upon being adopted by all participants in the contest - no small subgroup of individuals using a different standard of behavior can invade and "take over". The emphasis of the evolutionary approach is not on explaining actions (as a result of particular choice or otherwise), but on the diffusion of forms of behavior in groups (as a result of learning, imitation, reproduction or otherwise).

In a *finite* population of n individuals the smallest meaningful number of

deviationists is one. Starting from this observation Schaffer (1988) defined a finite population ESS as follows:

Definition 1: Let a strategy x be adapted by all players i, $i = 1, \ldots, n$.

i) A strategy $\bar{x} \neq x$ can invade x, if the pay-off for a single player using $\bar{x}$ (against $(n-1)$ players using x) is strictly higher than the pay-off of a player using x (against $(n-2)$ other players using x and the deviationist using $\bar{x}$).

ii) A strategy x^{ESS} is evolutionarily stable, if it cannot be invaded by any other strategy.

Formally, consider any expenditure level x; then non-invasibility of x in a finite population of n players requires that

$$\Pi_1(\bar{x}, x, \ldots, x) < \Pi_i(\bar{x}, x, \ldots, x) \text{ for all } \bar{x} \neq x \qquad (*)$$

and $i = 2, \ldots, n$. Player 1 deviates from x to $\bar{x}$, while players 2 to n stick with x. Note, that this means that the deviation is not profitable in *relative* terms. (Player 1's absolute utility $\Pi_1(\bar{x}, x, \ldots, x)$ might have increased over $\Pi_1(x, \ldots, x)$, but the other players' payoffs profit even more from 1's deviation). In biology x is termed an incumbent (strategy) and $(\bar{x})$ a mutant (strategy).

Definition 2: A strategy $x = x^{ESS}$ is an evolutionary stable strategy (ESS), if $\Pi_1(\bar{x}, x^{ESS}, \ldots, x^{ESS}) < \Pi_i(\bar{x}, x^{ESS}, \ldots, x^{ESS})$ for all $\bar{x} \neq x^{ESS}$ and $i = 2, \ldots, n$.

Hehenkamp et al. (2004) derive the following result on existence of ESS in Tullock contests for $r \leq \frac{n}{n-1}$.

Theorem 1:

i) *Individual expenditures and aggregate rent-dissipation in the unique evolutionary equilibrium $x = ((r/n)V, ..., (r/n)V)$ are always higher than in the unique efficient (Nash) equilibrium*
   $$x^* = (((n-1)/n^2)rV, ..., ((n-1)/n^2)rV).$$

ii) *Aggregate rent-dissipation in the unique evolutionary equilibrium is independent of the number of contestants, it is solely determined by the rent-seeking technology (contest success function) and the value of the rent:*
   $$n\frac{r}{n}V = rV$$

iii) *For $r > 1$ there is overdissipation of the rent in the unique evolutionary equilibrium; for $r = 1$ there is full dissipation of the rent and for $r < 1$ there is underdissipation of the rent.*

Hence ESS exists under precisely those conditons, which imply existence of a symmetric Nash equilibrium *in pure strategies*. In particular part (iii) of the Theorem attracted a great deal of attention: it demonstrates that overdissipation of the rent is *ex ante* compatible with pure strategy equilibrium, albeit not w.r.t. Nash equilibrium but evolutionary equilibrium ESS. Ex ante overdissipation is generally incompatible with Nash equilibrium (and individual rationality implied by it); however, there are some overdissipation results in the literature of Nash contest analysis that refer to *ex post* realisations of Nash equilibrium strategies , which do not imply overdissipation ex ante, i.e. in expected payoffs: Baye et al. (1999) show that overdissipation ex post

may occur incidentally as an outcome of a *mixed* strategy equilibrium in Tullock contests for $r > \frac{n}{n-1}$; i.e. when a symmetric equilibrium in pure strategies does not exist; but even in these cases ex ante dissipation is less or equal 100%. Similarly, the overdissipation results obtained by Lim and Matros (2009) and Fu et al. (2015) only apply incidentally *ex post*. They consider Tullock contests with a stochastic number of players, in which either each player enters with an exogenously fixed probability (Lim and Matros, 2009) or chooses a positive probability of entry (Fu et al., 2015). If then -incidentally- too many players have entered, overdissipation occurs in pure strategy equilibrium. But before realisations of the mixed entry strategies have occured *expected* rent dissipation is less than full rent dissipation.

Gu et al. (2019) examine both of these models from an evolutionary point of view and derive ex ante overdissipation results in ESS. Most notably, overdissipationcases exist for *all* $r \in (0, \frac{n}{n-1})$ in the case of endogeneous entry probabilities; i.e. convexity of the impact function is not required for overdissipation. In the following we will provide a new perspective on these overdissipation results, in particular the ones referring to ex ante overdissipation in evolutionary equilibrium, by giving an alternative Nash interpretation of ESS-behavior in contests.

3 Nash interpretations of evolutionary equilibrium ESS

a) Rational relative payoff maximizers

The standard interpretation of ESS behavior in games with finitely many players *as if* it emerged from Nash equilibrium was already given in the seminal contribution by Schaffer (1988).

The condition $(\ast)$ of non-invasibility is equivalent to the requirement that the difference

$$\Pi_1(\overline{x}, x^{ESS}, \ldots, x^{ESS}) - \Pi_i(\overline{x}, x^{ESS}, \ldots, x^{ESS}) \qquad i \in 2, \ldots, n$$

attains its maximal value 0 (as funtion of $\overline{x}$) at $\overline{x} = x^{ESS}$. In other words, if *all* players play x^{ESS} they do *not* behave as absolute payoff maximizers as the solution of the problem $\underset{\overline{x}}{Max} \; \Pi_1(\overline{x}, x^{ESS}, \ldots, x^{ESS})$ need not solve the problem

$$\underset{\overline{x}}{\text{Max}} \; \Pi_1(\overline{x}, x^{ESS}, \ldots, x^{ESS}) - \Pi_i(\overline{x}, x^{ESS}, \ldots, x^{ESS}) \quad \text{with } i \in 2, \ldots, n$$

In fact, the class of games for which this is the case is essentially restricted to zero-sum games, respectively generalisations of them like competitive games; see Guse et al. (2010)on the equivalence of Nash equilibrium and ESS. The above condition rather says that in an ESS-solution players behave *as if they were relative profit maximizers*. This is the essence of evolutionary pressure on fitness in nature that can produce 'spiteful' behaviour (Hamilton, 1970). Spiteful social behavior entails that the costs imposed onto the recipient are greater than the costs suffered by the actor, and this precisely works to the relative advantage of the actor. E.g. a contestant increases his effort beyond the level of the Nash equilibrium effort, because his loss in absolute payoff by doing so (recall that the Nash effort maximizes absolute payoff) is less than the loss of the opponents (due to decreased winning probability).

To make this argument more explict consider all players $i = 1, ..., n$ simultaneously with player m being the mutant. Write any strategy combination $x = (x_1, ..., x_n)$ as $x = (x_i, x_{-i})$ from player $i's$ point of view; x_i denotes i's own strategy and x_{-i} the $(n-1)$-vector of the other players' strategies. Then

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the payoff functions can be written as $\Pi_i = \Pi_i(x_i; x_{-i}), i = 1, ..., n$. So the mutant's payoff functions is given by $\Pi_m = \Pi_m(x_m, x_{-m})$. Since an ESS is symmetric by definition we can write the payoff of a player $i \neq m$ as the *average* of the payoffs to all players making up x_{-m} ; i.e.

$$\Pi_i = \Pi_{ESS} = \frac{1}{n-1} \cdot \sum_{i \neq m} \Pi_i(x_i, x_{-i})$$

The relative payoff maximization problem from above then becomes

$$\max_{x_m} \Pi_m = \Pi_m(x_m, x_{-m}) - \frac{1}{n-1} \sum_{i \neq m} \Pi_i(x_i, x_{-i}) \qquad \text{(RPM)}$$

and an ESS as defined in Definition 2 must be a symmetric Nash equlibrium of this 'beat-the average' game of Shubik and Levitan (1980), in which each player tries to realise a better result than the average result of all other players (see Schaffer 1989, p.38/39).

So we can state

Lemma 1: *An ESS strategy of a symmetric (contest) game with payoffs* $\Pi_i(x_i, x_{-i}), i = 1, ..., n$, *is identical to the symmetric Nash equilibrium strategy of the same (contest) game played by relative payoff maximizers with payoff functions* $\Pi_i(x_i, x_{-i}) - \frac{1}{n-1} \sum_{i \neq j} \Pi_j(x_j, x_{-j})$.

The relative payoff maximizers of the game in Lemma 1, in fact, play a zero-sum game as

$$\sum_{i=1}^{n} \left(\Pi_i(x_i, x_{-i}) - \frac{1}{n-1} \sum_{i \neq j} \Pi_j(x_j, x_{-j}) \right) = 0 \quad \text{for all } x = (x_1, ... x_n).$$

Hence defining a new relative payoff game with these relative payoff func-

tions as "absolute" primitives would not result in anything new. In a zero-sum game absolute and relative payoff maximization turn out to be identical (as the relative payoff equals $\frac{n}{n-1}$ times the absolute one).

Lemma 1 can be interpreted in the following way: for a relative payoff maximizer more is at stake than just winning V or not; because if he wins he improves his relative position with regard to all non-winners, and if he loses he worsens his relative position to the winner. More specifically, a somewhat 'loose' argument suggests that Π_i (according to RPM) increases by V, if i wins, whereas it decreases by $\frac{1}{n-1}V$ if he loses (as then the term $\sum_{i \neq j} \Pi_j(x_j, x_{-j})$ must increase by V, which gets weighted by the averaging factor $\frac{1}{n-1}$). The total stakes are hence $V - (-\frac{1}{n-1}V) = \frac{n}{n-1}V$. And consequently, an *absolute* payoff maximizer would have to face a rent of $\frac{n}{n-1}V$ to be confronted with the same stakes. This argument is made precise next from a more general perspective; namely, from the view of transfer contest theory. Transfer contests are a type of 'self-financing' contest, in which both, winning and losing, come with a 'prize'; winning with a positive one, losing with a negative one.

b) Transfer contests

We now specialize the general payoff fuctions $\Pi_i, i = 1, ..., n$, to those used in contest theory, i.e.

$$\Pi_i(x_1, ..., x_n) = p_i(x_1, ..., x_n) \cdot V - x_i \quad \text{for } i = 1, ..., n$$

with $\sum_{i=1}^{n} p_i(x_1, ..., x_n) = 1$ and $p_i(x_1, ..., x_1) \geq 0$ for all $(x_1, ..., x_n) \in \mathbb{R}_+^n$. $p_i(x_1, ..., x_n)$ denotes the probability of winning the contest for player i as a function of all efforts made by all contestants. The contested rent is valued at V by all contestants. For such contest games an alternative interpretation

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of ESS behaviour in terms of Nash equilibrium behaviour has been suggested by Leininger (2003), who notes that "contests and evolutionary ESS analysis...are both concepts that entered economics via biology"(p.178). In nature competition resp. contesting exists because there is nothing on earth which amounts to an infinite resource; everything is finite or limited. A prize or 'rent' appropriated from nature by an individual organism, human or otherwise, comes at the cost of the non-winners. This amounts to an "ecological" point of view as it factors in the changing ecological situation of the potential losers which calls for an evolutionary response in terms of adaptation. Evolutionarily stable behavior should reflect this response. Our next result says that this evolutionary response is exactly described by rational behavior of *absolute* payoff maximizers in a transfer contest vor V (as opposed to a direct contest for V).

The economic literature on rent-seeking has considered a particular type of contest, in which the losers have to provide the source of the gain for the winners. I.e. winning the prize V entails a transfer from the unsuccessful contestants to the winner in the size of V, the classical example is constituted by political contestability of rents when losers are "taxed" in the amount of V. Since in those contests a transfer (or tax) payment of an unsuccessful agent has to be made regardless of her expenditure decision, staying out of the contest would not guarantee a zero pay-off; he nonetheless would have to finance part of the transfer to the winning agent. E.g. protectionist policies transfer income from one group to another, and the losing group members suffer a loss irrespective of efforts to resist (or influence) those policies or not. Those contests have been termed *transfer contests* (see e.g. Appelbaum and Katz, 1986, and Hillman and Riley, 1989). In a transfer contest players make expenditures in order to secure a reward *and* in order to avoid a loss. In

the symmetric world of ESS analysis we can assume that "losses" would be distributed equally across non-winners. As a consequence for any contestant in nature more is at stake than just the value V of the rent, because if he does not win he loses the share $\frac{1}{n-1} \cdot V$ as "payment" to the winner. In such a contest the pay-off to player $i, i = 1, ..., n$ reads

$$p_i(x_1, ..., x_n) \cdot V + (1 - p_i(x_1, ..., x_n))(-\frac{V}{n-1}) - x_i. \qquad \text{(TC)}$$

It is (TC), which a contestant maximizes in *Nash* equilibrium of a transfer contest for V. What effectively is at stake for a contestant is the difference in payoffs between winning and losing and this difference now adds up to $V - (-\frac{1}{n-1} \cdot V) = \frac{n}{n-1}V$.

This can be seen directly from (TC) by simply rearranging terms in the objective function. (TC) is equivalent to

$$p_i(x_1, ..., x_n) \cdot (\frac{n}{n-1} \cdot V) - x_i - \frac{1}{n-1}V$$

which, in turn, is identical - except for the *constant* $-\frac{1}{n-1} \cdot V$ - to the payoff function of a contestant *in an (ordinary) contest for rent* $\bar{V} = \frac{n}{n-1}V$. We refer to $\bar{V}$ as the *transfer value* of V.

We then state

Lemma 2: *Nash behavior in a transfer contest for V is identical to Nash behavior in the contest for $\overline{V}$, the transfer value of V.*

We next show that the change in Nash equilibrium behavior of an absolute payoff maximizer, if a contest for V turns into a transfer contest for V, exactly

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leads to ESS behavior: Observe that we can write

$$1 - p_i(x_1, \ldots, x_n) = \sum_{j \neq i} p_j(x_1, \ldots, x_n)$$

and hence maximization of (TC) is seen to be equivalent to maximization of

$$p_i(x_1, \ldots, x_n) \cdot V - x_i - \frac{1}{n-1} \cdot \sum_{j \neq i} p_j(x_1, \ldots, x_n) \cdot V,$$

which in turn is equivalent to (RPM), if we set $i = m$ and add the constant $\frac{1}{n-1} \cdot \sum_{j=2}^{n} x_j$:

$$\max_{\bar{x}} p_1(\bar{x}, x_2, \ldots, x_n) \cdot V - \bar{x} - \frac{1}{n-1} \cdot \sum_{j=2}^{n} \left[p_j(\bar{x}, x_2, \ldots, x_n) \cdot V - x_j \right]$$

Hence it follows

Lemma 3: (Leininger, 2003) *Behavior in an evolutionary equilibrium ESS of a contest for V is identical to (rational) behavior in a symmetric Nash equilibrium of the corresponding* transfer *contest for V.*

Lemma 3 gives an exact measure for the degree of spite involved in evolutionarily stable behavior in a contest for V: a contestant *increases* expenditures (over the Nash level) up to the level he would choose (as a Nash player), if the contest were for the *transfer* value of V. Indeed, one can argue that the self-evident "ecological" quality of contests *in nature* implies that it is the transfer value of V, that is implicit in evolutionarily determined behavior (and not the nominal value of V). It is the former that correctly relates to survival, not the latter. This view has the advantage that it gives an *as if*-explanation of ESS behavior in terms of a Nash equilibrium between players

who are *absolute* payoff maximizers. Firstly, it allows to derive and compute ESS behavior - which in the "classcial" view is rationalized through the optimal Nash calculus of (non-standard) relative payoff maximizers (Schaffer, 1988) - from standard Nash calculus of absolute payoff maximizers; and secondly, it facilitates an objectification of the rent dissipation debate, in particular with regard to overdissipation, which is entirely rooted in absolute payoff considerations. In this regard Lemma 3 in connection with Lemma 1 in particular shows the correct equivalent to the behavior of relative payoff maximizers in a contest for V is behavior of absolute payoff maximizers in the *transfer* contest for the same V. We substantiate these claims in the next two sections.

4   Nash and ESS solutions

We are now exploring implications of the last two Lemmas. A first obvious implication of them is, that one can read off from the Nash equilibrium solution of a contest its ESS-solution. If the former is known for a rent V, it must be known for rent $\frac{n}{n-1} \cdot V$ as well and, hence, the evolutionary stable strategy ESS becomes known. It neither is necessary to do direct ESS calculations nor to perform relative pay off maximization.

Recall the results for Tullock contest from section 2:

Tullock's (1980) solution for Nash equilibrium is

$$x^{NE} = \frac{n-1}{n^2} \cdot r \cdot V$$

if we replace V by $\bar{V} = \frac{n}{n-1}V$, the transfer value of V, we get

$$\bar{x}^{NE} = \frac{n-1}{n^2} \cdot r \left(\frac{n}{n-1} V \right) = \frac{r}{n} \cdot V$$

which is the ESS solution derived from the ESS definition in Hehenkamp et al. (2004), see Theorem 1.

Our transfer contest interpretation gives a general relationship between Nash and ESS solution of contests:

$$X^{ESS} = \frac{n}{n-1} \cdot x^{NE}$$

and, consequently, for rent dissipation RD in these solution concepts

$$RD^{ESS} = \frac{n}{n-1} \cdot RD^{NE} = \frac{n}{n-1} \left(n \cdot \frac{n-1}{n^2} rV \right) = rV$$

Note again, that for $r > 1$ this implies overdissipation of V in ESS, but *not* overdissipation of the transfer value $\bar{V} = \frac{n}{n+1} \cdot V$ as $r \leq \frac{n}{n+1}$. Similarly , for Tullock contests with stochastic participation, in which each of n potential contestants only enters the contest with a fixed probability $p > 0$, so that the number of actual contestants is ex ante unknown to a contestant, Lim and Matros (2009) have shown, that for $r \leq \frac{n}{n-1}$ the Nash equilibrium entails $x^{NE} = r \cdot V \left[\sum_{i=1}^{n-1} C_i^{n-1} \cdot p^i (1-p)^{n-1-i} \cdot \frac{i}{(i+1)^2} \right]$ with $C_i^{n-1} = \frac{(n-1)!}{i!(n-i-1)!}$, whereas Gu et al. (2019) show in their Theorem 2, that this expression multiplied by the factor $\frac{n}{n-1}$ constitutes the ESS solution for any $r \leq \frac{n}{n-1}$.

Interestingly, this relationship between Nash an ESS solutions in contests holds beyond Tullock contest and pure strategy Nash equilibrium: the es-

sential substitution of $(1 - p_i)$ by $\sum_{j \neq i} p_j$ in (TC), which turned the transfer contest into the beat-the-average game, still holds for all-pay auctions:

Consider the (first-price) all pay auction, which is obtained as the limiting contest of the Tullock family for $r \to \infty$:

$$p_i(x_1, ..., x_n) = \begin{cases} 1 & \text{if } x_i > \max_{j \neq i}\{x_j\} \\ 0 & \text{if } x_i < \max_{j \neq i}\{x_j\} \\ \frac{1}{m} & \text{if there are } m \text{ maxima} \end{cases}$$

Hillman and Samet (1987) provided the symmetric Nash equilibrium solution for n contestants in mixed strategies by the following distribution function of bids in equlibrium:

$$F^*(x) = \left(\frac{x}{V}\right)^{\frac{1}{n-1}} \text{ on } [0, V];$$

i.e. for $n = 2$ both bidders would choose their bid from a uniform distribution on $[0, V]$. $F^*(x)$ denotes the probability that no other contestant's bid exceeds x (and the contestant considered hence wins). The expected bid of a contestant in equlibrium then is given by $E^*(x) = \frac{V}{n}$ and as a consequence full rent dissipation applies:

$$E^*(RD) = n \cdot E^*(x) = V$$

The ESS solution for the (first-price) all pay auction is then obtained from Hillman and Samet (1987) by using our two Lemmas for $\bar{V} = \frac{n}{n-1}V$:

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$$\bar{F}^*(x) = \left(\frac{x}{\bar{V}}\right)^{\frac{1}{n-1}} \quad \text{on } [0, \bar{V}] \text{ i.e.}$$

$$\bar{F}^*(x) = F^{ESS}(x) = \left(\frac{n-1}{n} \cdot \frac{x}{V}\right)^{\frac{1}{n-1}} \quad \text{on } \left[0, \frac{n}{n-1}V\right].$$

It can be checked directly (and much more elaborately as shown by Gu (2018)) that this, indeed, is the unique ESS. Note, in particular, that the support of $\bar{F}^*$ extends beyond V. Note also that the ESS strategy first-order stochastically dominates the Nash solution as

$$F^{ESS}(x) < F^*(x) \text{ for all } x \in (0, \bar{V}) \text{ (as } F^*(x) = 1 \text{ for all } x > V).$$

We can now compute the *expected* behavior under F^{ESS} as

$$E(x^{ESS}) = \int_0^{\bar{V}} x d\bar{F}^*(x) = \int_0^{\bar{V}} x(\bar{F}^*)'(x)dx$$

$$= \int_0^{\frac{n}{n-1}V} x\left(\frac{n-1}{n} \cdot \frac{1}{V}\right)^{\frac{1}{n-1}} \cdot \frac{1}{n-1} \cdot x^{-\frac{n-2}{n-1}} \, dx$$

$$= \frac{V}{n-1} = \frac{\bar{V}}{n}$$

and we see that

$$E(x^{ESS}) = \frac{V}{n-1} = \frac{n}{n-1} \cdot \frac{V}{n} = \frac{n}{n-1} \cdot E^*(x)$$

So the expected bid in ESS equilibrium equals $\frac{n}{n-1}$ times the expected bid in Nash equilibrium.

Analogously, the symmetric Nash equilibrium solution of the second-price all-pay auction under complete information, which is given in Vartiainen

(2007) as the mixed strategy

$$G^*(x) = \left(1 - e^{-\frac{x}{V}}\right)^{\frac{1}{n-1}} \text{ on } [0, \infty)$$

translates into the ESS-solution by replacing V with $\frac{n}{n-1}V$ as

$$G^{ESS}(x) = \left(1 - e^{-\frac{n-1}{n} \cdot \frac{x}{V}}\right)^{\frac{1}{n-1}} \text{ on } [0, \infty)$$

We are not aware of both of these all-pay auction ESS solutions in the existing literature.

5 Rent-dissipation in Nature: a Reassessment

The interpration of ESS behavior as a symmetric Nash equilibrium solution in a transfer contest is from an evolutionary point of view appealing because it captures the ecological view of a -locally or globally- closed system (like "environment" or "nature") containing the entity of all resources. There is simply no natural outside-world left that could account for further -*exogenously* given- resources. Hence any rent V up for contest in such a system must result in a *transfer* contest for V; i.e. V must come from the total stock of resources that could potentially become contestable (and consumable) among the contestants. Note that we do *not* assign (common) property rights in these resources. They just exist (and must be won) for potential consumption by any *one* agent. The definition of contest competition by Nicholson (see Introduction) clearly suggests that it is a zero-sum game in resources for survival at subsistence level; neither side can win without the other losing. Hence, if a particular contestant -the winner- obtains V from a contest, the group of losers experiences a reduction of V in the stock of

potentially consumable resources (just as the winner does), for which they *exclusively* have to share the burden; so each single loser accounts with a loss of $\frac{1}{n-1}V$ for it. Consequently, what is at stake in a contest for V in nature is $V - (-\frac{1}{n-1}V) = \frac{n}{n-1}V = \bar{V}$, the difference in payoffs between winning and losing. And this is precisely mirrored in the "aggressiveness" inherent to ESS behavior, which makes it non-invadible.

It follows that the evolutionary solution concept ESS behaves w.r.t. $\bar{V}$, the transfer value of V, *exactly* in the same way as the solution concept Nash equilibrium w.r.t. V. Since $\bar{V}$, the transfer value of V, is the correct "effective" value of the rent in ecological terms this sheds a new light on known results, in particular it qualifies the extensive "overdissipation"-debate as somewhat contrived.

Look, again, at the classical Tullock contest with parameter r: Nash equilibrium and ESS (in pure strategies) both exist if and only if $r \leq \frac{n}{n-1}$; let us consider two cases for a given n

   i) $r < \frac{n}{n-1}$, then $RD^{NE} = n \cdot x^{NE} = \frac{n-1}{n} \cdot rV < V$ and underdissipation of V in Nash equilibrium applies; and $RD^{ESS} = r \cdot V < \frac{n}{n-1}V = \bar{V}$ and underdissipation of $\bar{V}$ in ESS applies.

   ii) $r = \frac{n}{n-1}$, then $RD^{NE} = V$ and full dissipation of V applies; and $RD^{ESS} = r \cdot V = \frac{n}{n-1}V = \bar{V}$ and full dissipation of $\bar{V}$ applies.

This is summerized in Table 1.

Tullock parameter	NE	ESS
$r < \frac{n}{n-1}$	$\frac{n-1}{n} \cdot rV < V$	$r \cdot V < \frac{n}{n-1} \cdot V = \bar{V}$
	underdissipation of V	underdissipation of $\bar{V}$
$r = \frac{n}{n-1}$	V	$r \cdot V = \frac{n}{n-1} \cdot V = \bar{V}$
	full dissipation of V	full dissipation of $\bar{V}$

Table 1: Total rent dissipation in Nash equilibrium and ESS of Tullock contests.

Moreover, if we look at the influence of n for *any* given $r, r \leq \frac{n}{n-1}$, then we see that

$$\lim_{n \to \infty} RD^{NE} = \lim_{n \to \infty} (\frac{n-1}{n} rV) = r \cdot V = RD^{ESS}$$

(recall from Theorem 1 above that rent dissipation in an ESS is *independent* of the number of contestants). At the same time $\lim_{n \to \infty} \bar{V} = \lim_{n \to \infty} (\frac{n}{n-1} V) = V$, so in the limit transfer value and value of the rent *coincide*, as the contribution of a single loser in the transfer contest becomes negligible; while the range for (simultaneous) existence of Nash equilibrium and ESS in pure strategies shrinks to $\{r | r \leq 1\}$. And only in the boundary case of $r = 1$ does full rent-dissipation of the same rent $V = \bar{V}$ apply in both solutions.

To summarize, for increasing values of r Nash equilibrium underdissipates V less and less until value $r = \frac{n}{n-1}$ is reached, at which V gets fully dissipated. Further increases in r lead to non-existence of (symmetric) pure strategy equilibrium and full dissipation of V in symmetric mixed strategy equilibrium up to (and including) $r = \infty$, which is the all-pay auction. Completely analogously, for increasing values of r ESS underdissipates $\bar{V}$ less

and less until the value $r = \frac{n}{n-1}$ is reached, at which $\bar{V}$ gets fully dissipated. Further increases in r lead to non-existence of ESS in pure strategies and full dissipation of $\bar{V}$ in symmetric mixed strategy equilibrium up to (and including) the value $r = \infty$; which describes the all-pay auction.

We state

Lemma 4: *Rent dissipation in Nash equilibrium of a contest for rent V is limited by the value of the rent V; whereas rent dissipation in evolutionary equlibrium ESS of a contest for V is limited by $\bar{V}$, the transfer value of V; irrespective of the number of contestants and the nature of equilibrium as mixed or pure.*

As we have argued that contests in nature have to be understood as transfer contests among rational absolute payoff maximizers rather than contests between relative payoff maximizers Lemma 4 yields an impossibility result for the occurrence of overdissipation (of $\bar{V}$).

Theorem 2: *Ex ante overdissipation cannot occur in evolutionary equilibrium ESS of a contest.*

6   Conclusion

The interpretation of the ESS solution of a contest as the Nash equilibrium of the corresponding transfer contests identifies the effective rent at stake as its transfer value. We have then argued that the "biological" solution concept ESS from evolutionary game theory precisely accounts for this effective value of the rent, because contests in nature are transfer contests. The bottom line

then is: if we analyse a contest with the solution concept ESS, we analyse it as a transfer contest, in which the (transfer) value of the rent cannot get overdissipated. The straightforward relationship between value and transfer value allows us to compute ESS solutions, which refer to the transfer value, from Nash solutions, which refer to the value.

A final remark concerns the relevance of our reinterpretation of ESS (and relative payoff maximization) to the findings in the experimental literature on contests, in which overbidding and overdissipation is well documented (see Sheremeta (2013) and Dechenaux et al. (2015) for excellent surveys). Several studies show that overdissipation may be driven by spiteful preferences and relative payoff maximization (see e.g. Fonseca (2009); Eisenkopf and Teyssier (2013); and Mago et al. (2014)). The interpretation of the experimental data generated in these studies should get evaluated against the benchmark transfer value of the rent (not just value) to assess the degree of overdissipation (which generally would lower the experimentally measured degree of overdissipation).

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